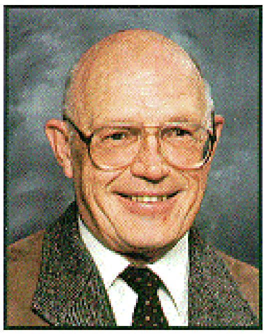
Historical and future views of rotor dynamics



by Donald E. Bently
Chairman and Chief Executive Officer
Bently Nevada Corporation
President, Bently Rotor Dynamics Research Corporation

Nomenclature

- c = Bearing radial clearance
- D = Fluid bearing damping
- D_s = Rotor system structural and external damping
- F_y = Force in + y direction
- K_d = Fluid film Direct Stiffness
- K_q = Fluid film Quadrature Stiffness
- K_{eff} = Rotor effective (modal) stiffness
- M_{eff} = Rotor effective (modal) mass
- x = Distance in x direction
- y = Distance in y direction
- λ = Fluid average circumferential velocity ratio
- Ψ = Shaft attitude angle
- $\dot{\psi}$ = Angular velocity of rotor precession (Lomakin)
- Ω = Rotor rotative speed
- ω = Angular velocity of rotor precession
- ω_n = Rotor system natural frequency
- W = Rotor weight

There have been many workers in machinery vibration and rotor dynamics who have contributed to the advancement of the field. I can easily name several people in industry or in academia, in many countries of the world, that I look to as my senior in capability.

Many of what seem to be "new" inventions are really developments of work done by others. Typically, Orbits were done by Newkirk the year I was born. Polar plots were done by Bishop, and probably others, on machinery resonances in the early 60's. In fact, Polar plots were quite popular for control system work, the area in which I specialized, long before that. The Keyphasor® was used by Stone and Underwood in their oil whip work in the mid 40's. The Digital Vector Filter does mathematically the fundamental job of convolution that Thearle introduced in his excellent balancing work from the 30's. I could continue this list, but perhaps the point is clear.

In the future, many more changes and improvements in machinery and in the observation of this machinery will occur. Totally sealed machines are now being built and installed. Machines with better efficiency, longer life, lower cost, and higher performance are certain to evolve. The science of the measurement of vibration must also evolve.

It is worthwhile to reexamine some of the more interesting cases in the history of rotor dynamics. This article examines a number of early developments and concepts in rotor dynamics. The discussion will include the Half-Circle Assumption of shaft deflection versus load, bearing lubrication pressure versus rotor stability, rotor mass and stability, the use of root locus techniques in stability analysis, and radial stiffness of fluid-film bearings at zero eccentricity. Some of these concepts are, in the light of our present day knowledge, known to be incomplete. Others can be seen to have been very forward-looking for their time and can still provide useful guidance on potential directions for future research. Still others are actively being researched at this time.

The Half-Circle Assumption of shaft displacement versus load

One of the most amusing and unfortunate assumptions made in the early days of rotor dynamics is that the plot of shaft position versus applied load takes the form of a semicircle. This semicircle is very crudely (topologically) correct; however, it is the use of the circle to relate the fluid-film direct stiffness to quadrature stiffness that is the problem.

Figure 1 shows the load versus displacement assumption.

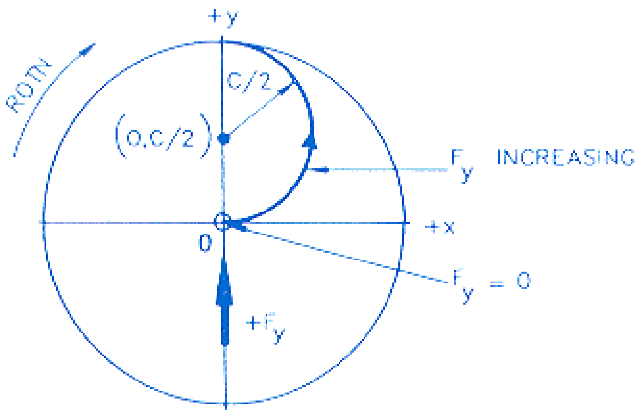


Figure 1
Shaft displacement locus inside a cylindrical bearing using the Half-Circle Assumption.

The equation of the assumed half circle is:

$$x = \sqrt{\left(\frac{c}{2}\right)^2 - \left(y - \frac{c}{2}\right)^2} = \sqrt{cy - y^2} \qquad \textit{(Note assumption)}\ (1)$$

where c is the radial clearance, and x and y are the displacements in the directions of the coordinate axes. Equation (1) reduces to:

$$x^2 + y^2 = cy \qquad \textit{(Note assumption)}\ (2)$$

The force equations for this simple system are, for y into x (clockwise) rotation (as shown in Figure 1):

$$F_y = K_d y + K_q x \qquad (3)$$

$$0 = K_d x - K_q y \qquad (4)$$

so that,

$$K_d = \frac{F_y y}{x^2 + y^2} \qquad (5)$$

$$K_q = \frac{F_y x}{x^2 + y^2} \qquad (6)$$

Taking the ratio of Equation (6) to Equation (5),

$$\frac{K_q}{K_d} = \frac{x}{y} \qquad (7)$$

Relating Equation (5) and Equation (6) back to the assumed semicircle relationship in Equation (1), these amusing and unreal relationships are obtained:

$$K_d = \frac{F_y}{c} \qquad \textit{(Note assumption)}\ (8)$$

$$K_q = \frac{F_y \sqrt{cy - y^2}}{cy} \qquad \textit{(Note assumption)}\ (9)$$

The attitude angle, Ψ, is actually:

$$\Psi = \arctan\left(\frac{x}{y}\right) \qquad (10)$$

Therefore, by Equation (7),

$$\Psi = \arctan\left(\frac{K_q}{K_d}\right) \qquad (11)$$

Obviously, the actual relationship between x and y is absolute as a function of clearance, c. However, with the Half-Circle Assumption, while the rigid relationship between direct stiffness, K_d, and quadrature stiffness, K_q, are now forced into the form:

$$\frac{K_q}{K_d} = \frac{x}{y} = \sqrt{\frac{cy - y^2}{y^2}} = \sqrt{\frac{c - y}{y}} \qquad \textit{(Note assumption)}\ (12)$$

or

$$\frac{K_d}{K_q} = \sqrt{\frac{y}{c - y}} \qquad \textit{(Note assumption)}\ (13)$$

there is nothing to establish the absolute value of either. It is well-known that, for constant rotative speed and fluid viscosity, the lubricant wedge support stiffness (i.e. the quadrature stiffness) is essentially constant at low through medium eccentricities, so that a plot of K_d and K_q may be made, as in Figure 2.

As a typical case, let

$$c = .01 \text{ inch} \qquad (14)$$

$$K_q = 10{,}000 \text{ lb/in} \qquad (15)$$

then, using Equation (13),

$$K_d = 10000\left(\frac{y}{.01 - y}\right)^{1/2} \text{ lb/in} \qquad \textit{(Note assumption)}\ (16)$$

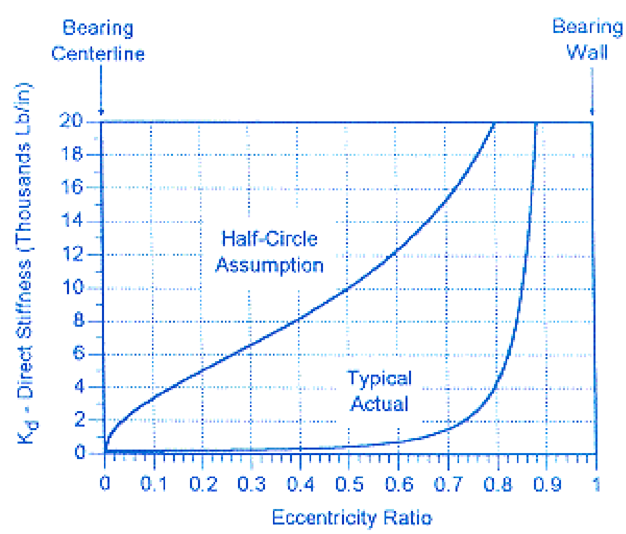


Figure 2
Bearing Direct Stiffness versus eccentricity ratio comparing typical actual behavior to the Half-Circle Assumption.

and, using Equation (1),

$$x = [(.01 - y)y]^{1/2} \text{ in} \qquad \textit{(Note assumption) (17)}$$

Finally, using Equation (8),

$$F_y = .01 K_d \text{ lb} \qquad \textit{(Note assumption) (18)}$$

The Half-Circle Assumption does not assume that K_q is constant; this additional assumption was made here to make the K_d term look something like its usual nonlinear shape.

In fact, however, the quadrature stiffness, K_q, is shown by both theory and experiment to be remarkably constant for low and medium eccentricity ranges, considering the natural nonlinearity and nonsymmetry of typical bearings at constant rotative speed, Ω.

Whereas the Half-Circle Assumption can yield a reasonable set of K_d and K_q terms, the vital error of their relationship is that the direct dynamic stiffness, K_d, is reasonably independent of rotative speed, Ω. A second error is to assume that the quadrature dynamic stiffness, K_q, is directly proportional to the rotative speed.

Two secondary errors of the Half-Circle Assumption are that (1) the assumed semicircle requires zero direct stiffness at the centerline, whereas an externally pressurized bearing has high direct stiffness at zero eccentricity, and (2) the fluidic inertial effect term may have the exact opposite effect, yielding a negative direct stiffness at low to medium eccentricities. These effects are shown typically in Figure 3.

The real typical steady displacement versus steady load for constant viscosity and various rotative speeds is shown in Figure 4. As can be observed, the semicircle is approximately correct for just one set of conditions, in the same manner that a stopped watch is correct twice a day.

One can conclude that the Half-Circle Assumption can be used only for an extremely crude representation of bearing behavior. The use of the equation for the semicircular relationship in serious rotor dynamics, however, leads to very poor results and continued misunderstanding of bearing and seal behavior.

The Lomakin Effect

Professor A. A. Lomakin wrote a very good paper on Pump Stability in 1958. It was his belief, so far as I can interpret, that pumps have a quadrature dynamic stiffness which increases with the square of speed. Our more recent work shows an increase of quadrature dynamic stiffness which is linear (power of one) with speed. Thus, from Lomakin (1958) (our interpretation),

$$\left(\frac{\Omega}{2} - \dot{\psi}\right)^2 \qquad \textit{(From Lomakin) (19)}$$

where Ω is the shaft rotative speed, and $\dot{\psi}$ is the angular velocity of rotor precession. However, we have found (Bently & Muszynska (1988)) that:

$$K_q = D(\omega - \lambda\Omega) + D_s\omega \qquad \textit{(20)}$$

where D is the damping of the fluid bearing, ω is the angular velocity of rotor precession, λ is the fluid average circumferential velocity ratio, and D_s is the rotor structural and external damping.

Recent papers on pumps with quotations of that work seem to infer that the direct stiffness of a pump increases with the square of speed. Surely, there could be such a direct stiffness

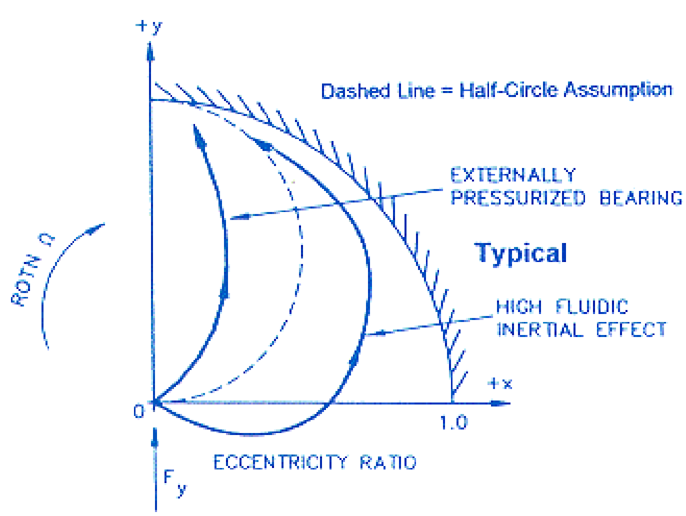


Figure 3
Typical shaft locus of static displacement versus static load showing the effects of external pressure and high fluidic inertial effect.

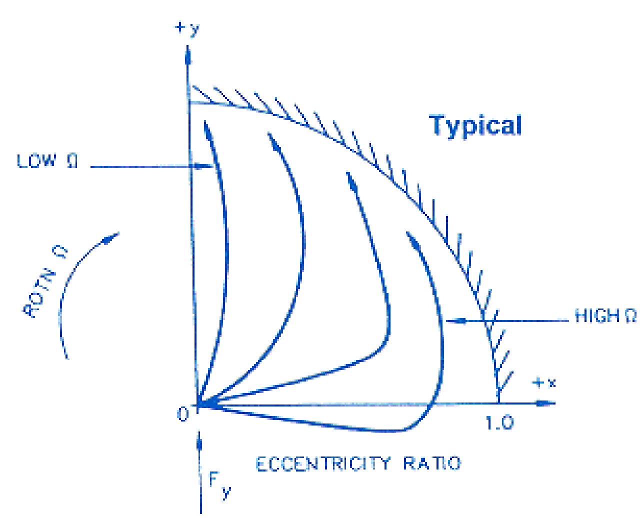
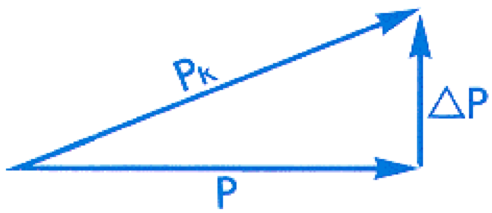


Figure 4
Typical shaft static displacement versus static load showing the effect of various rotative speeds.

term, especially if the pump pressure increased with the square of speed (which it tends to do), and that pressure increased the flow of a seal, bearing, or other rotor-stator interface. However, this is, in my belief, not much related to Lomakin.

It is my guess that it was Lomakin himself who confused the issue. Near the end of the paper, he notes (as translated),

> ***The frequency of the vibration under operational conditions and the combined action of elastic and hydrodynamic forces is:***



$P_k = \sqrt{P^2 + \Delta P^2}$ *(Note: Lomakin Statement)*

It is my observation that these frequencies do not combine in this fashion. Furthermore, it may be seen that his elastic term, P, is clearly a direct term; therefore, the hydrodynamic term, ΔP, (as evidenced by the Pythagorean vector addition), is a tangential (quadrature) term.

Studies undertaken at Bently Rotor Dynamics Research Corporation have shown the existence of a term which we call the fluidic inertia effect, because it varies with the square of the differential speed, as does Lomakin's expression. However, this effect is negative, not positive, and appears only for low and medium eccentricities and has no noticeable effect on stability, either mathematically or in tests.

Bearing lubrication pressure

As any technology develops, it occasionally gets off on a wrong track. Usually, these incorrect paths are not "bloopers," because at the time that they are done they represent good work. The problem usually occurs when all followers, in theory and in practice, continue to employ the incorrect line, even after it has been clearly replaced by a much better set of rules.

Very typical of this is the use of low oil pressure to feed sleeve bearings. It is, even now, repeatedly stated in specifications and recommended practices that supply lubricant pressure be limited, typically to 25 lbs/in^2. The use of very low oil pressure helps to assure that the bearing does *not* have lubricant a full 360 degrees around. This tends to make it much more difficult for the rotor system to go unstable. This technique has been well-known since Harrison (1919) stated that, "a fully lubricated bearing is always unstable." This is, in fact, often correct if nothing is done to correct the situation.

However, with proper bearing design, a high-pressure lubricated bearing is far superior to a low-pressure fed bearing and may have much higher efficiency, much better heat transfer, and much better load bearing characteristics. By using proper antiswirl and pressurized bearing design, much higher (and better controlled) stability characteristics can be obtained.

We have repeatedly shown that the first two cures for fluid-induced rotor system instability sources at bearings are:

1. Antiswirl
2. Increased lubrication pressure

Therefore, this restriction on use of high pressure lubricants represents a major impediment to progress on rotating machinery.

Rotor mass and stability

About twenty years ago, it was popular to suggest that, if a horizontal rotor system had fluid-induced instability, the rotor was too light for its bearing clearance. It has been often published that, to cure the instability, more weight must be added to the rotor. This rule seems to have quietly (and fortunately) disappeared, because adding rotor weight on a horizontal machine may have any of three results:

1. The fluid instability can become worse.
2. The effect on stability is indifferent.
3. The rotor system can be stabilized.

This is because added weight affects both the numerator and denominator of the basic natural frequency equation,

$$\omega_n = \sqrt{\frac{K_{eff}}{M_{eff}}} \qquad (21)$$

where ω_n is the natural frequency of the rotor system, K_{eff} is the effective (modal) stiffness of the rotor system, and M_{eff} is the effective (modal) mass of the rotor system.

The effect of the denominator is obvious; if the rotor weight, W, goes up, M_{eff} goes up proportionally, and the resonant frequency goes down, which is bad for stability.

The effect of the numerator is also fairly obvious. The direct spring coefficient of a bearing is highly nonlinear with eccentricity, increasing steeply at high eccentricity, and the direct spring is one of the components of the numerator. Thus, when K_{eff} (numerator) changes are directly proportional to M_{eff} (denominator) changes, the effect is indifferent.

If K_{eff} grows more rapidly than M_{eff}, then stability is enhanced.

However, when adding rotor mass to a horizontal machine, very often M_{eff} grows faster than K_{eff}, so that the fluid instability is worsened by the increased rotor weight.

It is obvious that the original proponents of this method of stability control recognized the distinction between adding rotor mass and adding a steady radial sideload. Perhaps later authors lost track of this simple distinction. Steady sideload is often a good stability enhancement procedure.

It is generally believed that the steady radial sideload provided by gravity in horizontal machines is **always** desirable. However, this sideload can lead to several possible problems. First, a steady sideload produces a cyclic rotative speed stress on the shaft which may contribute to shaft cracking. Second, when the shaft has asymmetry and a resonance at or near twice rotative speed, each time the stiffer axis of the shaft passes the top dead center location of the sideload, the resultant snapping action will strongly contribute to shaft fatigue and eventual failure.

Third, much energy may be wasted as heat at the high eccentricity spot in the bearing. Lastly, as noted by Kimball (1925), Internal Friction Forward Circular Whip Instability is encouraged by any deflection; therefore, the radial sideload cure for instability may also be a cause of instability. Some hydrogen and oxygen pumps probably suffer from this problem.

Lund and root locus

On occasion, proper theory and presentation has been developed and only a few pay attention. One example of this is the root locus graph done by Jorgen Lund (1974). Lund learned from Mel Prohl, and those two are considered by many, including myself, to be the major leaders of modern rotor dynamics. Figure 5 shows one of the root locus plots presented in Lund's paper.

This is a root locus plot as pioneered by Walt Evans (1954), who taught me root locus for control system design. Unfortunately, there are only a few other researchers (Hahn, E. Kramer, and T. Iwatsubo are among them) who have implemented this method of stability analysis and followed Lund's lead.

The root locus is the locus of a family of points, each of which represents the roots (eigenvalues) of the characteristic equation formed from the set of differential equations of the rotor system. These loci can be plotted in a plane representing the "real" and "imaginary" parts of the roots. (There is nothing "imaginary" about these values! They represent real, physical machine behavior. Better names for these terms are Decay/Growth Rate and Precession Frequency, respectively.)

The stability threshold (or threshold of instability) is reached when the Decay/Growth Rate is equal to zero. Once this boundary is crossed, the roots, when based on linear differential equations, become meaningless; the machine whirl or whip amplitude becomes bounded by nonlinearities in the fluid bearing or seal.

Since rotor system models are based on a large number of design parameters, such as masses, damping coefficients, stiffnesses, rotor rotative speed, fluid circumferential velocity ratios, and more, any parameter of interest can be varied in a systematic way to explore the effect of that parameter on stability.

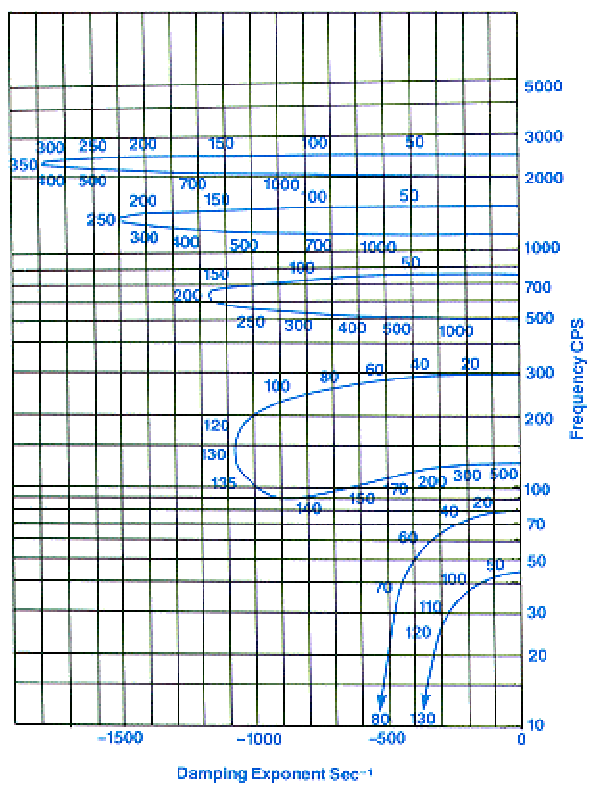


Figure 5

Root locus plot as presented by Lund (1974). The plot displays damped natural frequencies (vertical axis) versus damping exponent (horizontal axis). The numbers on the curves represent the bearing damping coefficients that were varied to analyze the stability of the rotor system he was studying.

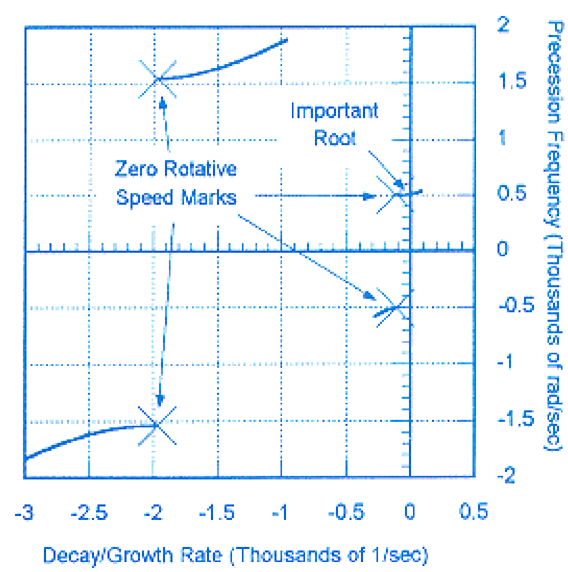


Figure 6

Root locus plot for a typical multi-DOF rotor system model showing the two lowest modes (both positive and negative roots for each mode). The loci represent rotative speed variation from 0 (marked by the X) to 2000 rad/sec. Only one root poses a threat to machine stability because it crosses the vertical axis.

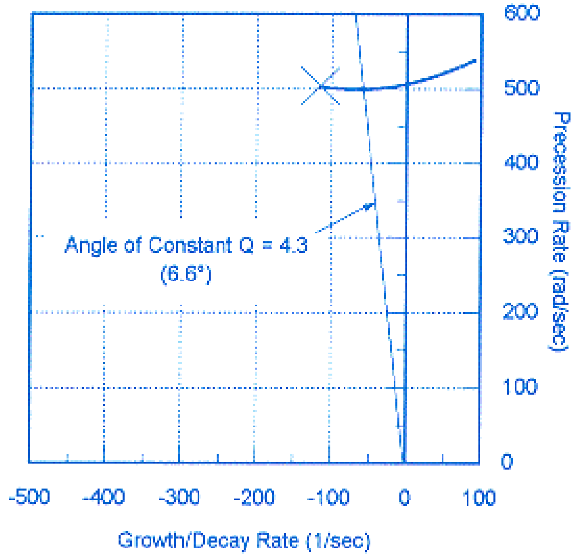


Figure 7
An enlarged view of the important root shown in Figure 6.

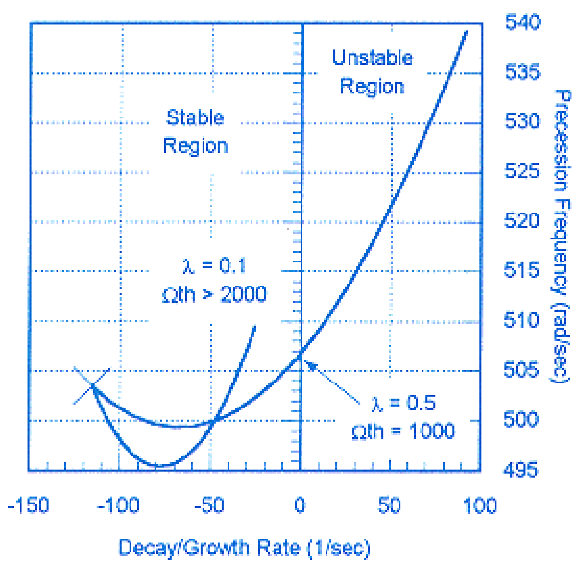


Figure 8
Root locus plot showing the effect of antiswirl on the important root in Figure 7. Both curves reflect changing rotative speed Ω (0 to 2000 rad/sec) for $\lambda = 0.5$ and $\lambda = 0.1$. The instability threshold, Ωth, is about 1000 rad/sec without antiswirl ($\lambda = 0.5$). With antiswirl ($\lambda = 0.1$) the instability threshold is increased to greater than 2000 rad/sec.

A typical example of such a root locus plot is shown in Figure 6. This graph was created using a multi-DOF, 2-mass rotor model which resulted in a sixth-order characteristic equation with complex coefficients, and the equation was solved numerically. The rotor rotative speed, Ω, was varied from 0 to 2000 rad/sec. Only the roots corresponding to the lowest two modes are shown. Note that they appear in conjugate pairs at zero rotative speed (at any other rotative speed, the roots are no longer conjugate).

Note also that only one of the four roots shown poses a threat to stability and that the nearly conjugate root (reverse precession) root moves toward greater stability. This is almost always the case; the first mode forward precession root is usually the only one that poses a stability problem. Figure 7 shows an enlarged view of the important root that was shown in Figure 6.

Root locus techniques can be used to study the effects of changing design variables on the stability and damping factor of a machine at the design stage and later to evaluate fixes to stability problems discovered during machine operation.

Figure 8 shows the results of an antiswirl study on the root shown in Figure 7. The rotor speed was again varied from 0 to 2000 rad/sec. The fluid average circumferential velocity ratio, λ, was varied from 0.5 to 0.1. The dramatic improvement in stability is evident in the figure. The machine, which was going unstable at a rotative speed of around 1000 rad/sec, is now stable at a rotative speed of over 2000 rad/sec.

The root locus is a powerful tool for stability control and will be used heavily in the future. Bently Rotor Dynamics Research Corporation is actively researching the application of root locus techniques to the solution of rotor dynamics stability problems.

Poritsky and radial stiffness

Another case of a man ahead of his time is that of H. H. Poritsky. He did the basic equations of stability very well for his time. He, as well as everyone else, used 1/2 as the lubricating fluid average circumferential velocity ratio which, although too restrictive, was not fundamentally wrong. However, his excellent work (Poritsky 1953) was heavily criticized because he included a radial spring at zero eccentricity. Apparently, other researchers claimed he was incorrect on this point, thereby disregarding his entire contribution. It is true that it is possible to have zero radial spring stiffness at zero eccentricity. I have built such bearings many times, and, of course, because the resulting mechanical resonance is at zero speed, they allow the machine to go into oil whirl at zero rotative speed.

However, most ordinary bearings do have at least a small radial stiffness at zero eccentricity, or alternatively, the steady state rotor position is slightly eccentric, or the shaft or seals supply a direct spring component. Thus, Poritsky's work was actually extremely useful.

Conclusion

History can be useful and interesting, but I do not believe in steering a boat by watching the wake. This is a partial review of things that went wrong, things that went right, and some

ideas about what will occur in the future. This study of "land mines" was made as an adjunct of another study which will be the genesis and antecedents of the Quadrature Dynamic Stiffness term published by Dr. Agnes Muszynska and myself,

$$K_q = (\omega - \lambda\Omega)D. \qquad (22)$$

I am pleased that it has a genesis, because it has been extremely difficult to get this paradigm shift into general acceptance.

Lack of knowledge and lack of measurement of these dynamic stiffness parameters represent a major handicap to the rotating machinery industry. This is because, without them, it is impossible to measure and specify the performance characteristics necessary to prevent instability.

This article shows some of the history of rotor dynamics in a form, hopefully, that will be useful to others in their pursuit of future improvements to our knowledge and control of machinery behavior. ■

Editor's Note: The December 1993 Orbit will include an article written by Donald Bently on root locus.

References

1. Bently, D. E., and Muszynska, A., "Role of Circumferential Flow in the Stability of Fluid-Handling Machine Rotors," Fifth Workshop on Rotordynamics Instability in High Performance Turbomachinery, Texas A&M University, College Station, Texas, May 1988.
2. Evans, Walter R., "Control-System Dynamics," McGraw-Hill, New York, NY, 1954.
3. Harrison, W. J., "The Hydrodynamical Theory of the Lubrication of a Cylindrical Bearing Under Variable Load, and of a Pivot Bearing," Transactions of the Edinburgh Philosophical Society, Edinburgh, Scotland, Vol. 22, 1919.
4. Kimball, A. L., "Internal Friction as a Cause of Shaft Whirling," Phil. Mag., VI, Vol. 49 (1925).
5. Lomakin, A. A., "Calculations of Critical Number of Resolutions and the Conditions Necessary for Dynamical Stability of Rotors in High-Pressure Hydraulic Machines When Taking Into Account Forces Originating in Sealings," (In Russian) "Energomashinostroyeniye," April 1958.
6. Lund, J. W., "Stability and Damped Critical Speeds of a Flexible Rotor in Fluid-Film Bearings," ASME Journal of Engineering for Industry, May 1974, p. 510.
7. Poritsky, H., "Contribution to the Theory of Oil Whip," Transactions of the ASME, August 1953, pp. 1153-1161.

Shaft CenterLINES

The ratio of 2X to 1X vibration - *A shaft crack detection myth*

by Malcolm Werner
Corporate Manager, MES
Bently Nevada Corporation

For many years, a myth has circulated in industry that relates the presence of and/or growth of a shaft crack to the ratio of two times synchronous vibration amplitude (2X) to a synchronous vibration amplitude (1X):

$$\text{Crack Ratio} = \frac{\text{2X Amplitude}}{\text{1X Amplitude}}$$

The myth states that a crack is present or a crack is growing when this ratio is increasing. If the ratio is constant or if there is no 2X, then there isn't a shaft crack, according to this myth.

Bently Rotor Dynamics Research Corporation (BRDRC) has been "keeping score" on shaft cracks for more than ten years. Based on this "score card" of actual shaft crack cases, if you follow the **crack ratio** formula, you will be **exactly wrong more than 75% of the time.**

BRDRC, Bently Nevada's Machinery Diagnostics Services, and our customers, have been using a field-proven shaft crack diagnostic methodology for many years. We have found that, in more than 75% of documented cases, the key indicator of shaft cracks is change in 1X amplitude or 1X phase lag angle. While 2X vibration is helpful in crack diagnostics, in more than 25% of cases there was no 2X vibration.

In addition, there are other sources for 2X vibration, such as the effect of a steady sideload into a nonlinear stiffness bearing or seal when there is some 1X bow. Therefore, the crack ratio can not only fail to predict a crack, it can give false indication of a crack.

The "score card" summarizes the key shaft crack indicators, based on experience. ■

Vibration score card for shaft cracks

Changing parameter	Prevailing indicator in shaft crack cases
1X Amplitude	75% of cases. Nearly always increasing amplitude
1X Phase	Often seen with 1X amplitude, probably rare by itself. Can be leading, lagging or constant phase.
2X Amplitude	25% of cases. Either 2X Amplitude (usually increasing) or Amplitude and Phase changes, depending on rotor system and crack properties.
2X Phase	